

COMPUTATIONAL METHODS AND IMPLEMENTATION TECHNIQUES FOR
PROBLEMS RELATED TO NAVIGATION, GUIDANCE, AND CONTROL

FINAL REPORT on NASA Grant ~~NGR~~ 22-007-068

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Summary of Work Accomplished on the Grant.

The work performed on this grant can be divided into the following areas.

1. Computational Methods for Optimization.

- 1.1. Differential Dynamic Programming [12, 27]
- 1.2. Conversational Language for Optimal Synthesis Experimenter (CLOSE) [17, 11]
- 1.3. Stochastic Optimization [10, 2]
- 1.4. Penalty Function Method for SVIC [18]
- 1.5. Parameter Optimization Methods [28, 8]

2. Control Theory.

- 2.1. New Result on SVIC [18, 26]
- 2.2. Singular Control Problems and New Necessary Conditions [25, 21]
- 2.3. Nonlinear Controllability [24]
- 2.4. Differential Games and Generalized Control Theory [20, 19, 22, 23, 15]

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3. Estimation and Decision Theory.

- 3.1. Nonlinear Filtering [16]
- 3.2. Smoothing for Correlated Noise [6, 7]
- 3.3. Pattern Recognition Problems [5, 13]

4. Applications.

- 4.1. Strapdown Navigation [16]
- 4.2. Maximum Range Aircraft [3, 1]
- 4.3. Re-entry Problems [4]
- 4.4. VSTOL Aircraft [9]

A total of 13 Ph.D. Theses were completed under the partial support of this Grant.

List of Publications.

- 1. D.H. Winfield, A.E. Bryson, Jr., "Nonlinear Feedback Solution for Minimum-Time Injection into Circular Orbit with Constant Thrust Acceleration Magnitude," Cruft Laboratory Technical Report No. 507, July 1966.
- 2. Y.C. Ho and P.M. Newbold, "A Descent Algorithm for Constrained Stochastic Extrema," Technical Report No. 524, May 1967; also in the Journal of Optimization Theory and Applications, Vol. 1, No. 3, November 1967, pages 215-231.
- 3. J.L. Speyer, R.K. Mehra, and A.E. Bryson, Jr., "The Separate Computation of Arcs for Optimal Flight Paths with State Variable Inequality Constraints," Cruft Laboratory Technical Report No. 526, May 1967; also presented at the Colloquium on Space Flight Optimization, University of Liege, Liege, Belgium, June 1967.
- 4. J.L. Speyer and A.E. Bryson, Jr., "A Neighboring Optimum Feedback Control Scheme Based on Estimated Time-to-Go with Application to Re-entry Flight Paths," Cruft Laboratory Technical Report No. 527, June 1967; also presented at the Colloquium on Space Flight Optimization, University of Liege, Liege, Belgium, June 1967.
- 5. P.M. Newbold and Y.C. Ho, "Detection of Changes in Characteristics of a Gauss-Markov Process," Cruft Laboratory Technical Report No. 531, June 1967; also IEEE Transactions in Aerospace Electronics, September 1968, Vol. AES-4, pages 707-718.
- 6. R.K. Mehra and A.E. Bryson, Jr., "Smoothing for Time-Varying Systems Using Measurements Containing Colored Noise," Technical Report No. 1, June 1967; also IEEE Transactions on Automatic Control, October 1968, pages 496-503.
- 7. A.E. Bryson, Jr. and L.J. Henrikson, "Estimation Using Sampled-Data Containing Sequentially Correlated Noise," Cruft Laboratory No. 533, June 1967; also presented at the AIAA Guidance, Control and Flight Dynamics Conference, August 14-16, 1967, Huntsville, Alabama, Proceedings Paper No. 67-541.

8. D.H. Winfield, "Function Minimization Without Derivatives by a Sequence of Quadratic Programming Problem," Technical Report No. 537, August 1967.
9. R.K. Mehra and A.E. Bryson, Jr., "Conjugate Gradient Methods with an Application to V/STOL Flight-Path Optimization," Technical Report No. 543, November 1967.
10. P.M. Newbold, "A Stochastic Approximation Scheme with Accelerated Convergence Properties," Technical Report No. 545, October 1967.
11. P.M. Newbold and A.K. Agrawala, "Two Conversational Languages for Control Theoretical Computations in the Time Sharing Mode," Technical Report No. 546, November 1967.
12. D.H. Jacobson, "New Second-Order and First-Order Algorithms for Determining Optimal Control: A Differential Dynamic Programming Approach," Harvard University Technical Report No. 551, Division of Engineering and Applied Physics, February 1968; also Journal of Optimization Theory and Applications, Vol. 2, No. 6, 1968.
13. A.K. Agrawala and Y.C. Ho, "On Pattern Classification Algorithms - Introduction and Survey," Technical Report 557, March 1968; also in Proceedings of IEEE, December 1968; also invited address at 1968 Delft International Workshop in Pattern Recognition.
14. A.W. Starr and Y.C. Ho, "Nonzero-Sum Differential Games," Journal of Optimization Theory and Applications, Vol. 3, No. 3, March 1969, pages 184-206.
15. Y.C. Ho and R.D. Behn, "On a Class of Linear Stochastic Differential Games," IEEE Transactions on Automatic Control, Vol. AC-13, June 1968, pages 227-240.
16. J.S. Lee and Y.C. Ho, "Analysis and Design of Integration Formulas for a Random Integrator," Technical Report No. 2, Division of Engineering and Applied Physics, August 1968.
17. P.M. Newbold, "The Matrix Algebra Program: A Conversational Language for Numerical Matrix Operations - Part II: Reference Manual," Technical Report No. 562, June 1968, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts.
18. D.H. Jacobson and M.M. Lele, "A Transformation Technique for Optimal Control Problems with a State Variable Inequality Constraint," Harvard University Technical Report No. 574, October 1968, IEEE Transactions on Automatic Control, October 1969.
19. W.W. Willman, "Formal Solutions for a Class of Stochastic Pursuit-Evasion Games," Technical Report No. 575, November 1968, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts; also Transactions of IEEE Group on Automatic Control, Vol. AC-14, No. 5, 1969, pages 504-510.

20. A.W. Starr and Y.C. Ho, "Further Properties of Nonzero-Sum Differential Games," Technical Report No. 577, November 1968, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts; also Journal of Optimization Theory and Applications, Vol. 3, No. 4, April 1969, pages 207-219.
21. D.H. Jacobson, S.B. Gershwin, and M.M. Lele, "Computation of Optimal Singular Controls," Harvard University Technical Report No. 580, January 1969; also IEEE Transactions on Automatic Control, February 1970.
22. W.W. Willman, "On a Class of Stochastic Pursuit-Evasion Games," Technical Report No. 585, April 1969, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts.
23. A.W. Starr, "Nonzero-Sum Differential Games: Concepts and Models," June 1969, Technical Report No. 590, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts.
24. S.B. Gershwin and D.H. Jacobson, "A Controllability Theory for Nonlinear Systems with Applications," Harvard University Technical Report 592, July 1969.
25. D.H. Jacobson, "Sufficient Conditions for Non-Negativity of the Second Variation in Singular and Nonsingular Control Problems," Technical Report No. 596, August 1969, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts, SIAM J. Control, to appear.
26. D.H. Jacobson, M.M. Lele, and J.L. Speyer, "New Necessary Conditions of Optimality for Control Problems with State-Variable Inequality Constraints," August 1969, Technical Report No. 597, Division of Engineering and Applied Physics, Harvard University, Cambridge, Mass.
27. D.H. Jacobson, "Differential Dynamic Programming Method for Solving Bang-Bang Control Problems: Algorithms and Error Analysis," October 1969, Technical Report No. 598, Division of Engineering and Applied Physics, Cambridge, Massachusetts IEEE Trans. Automatic Control, Dec. 1968.
28. D.H. Winfield, "A Fractional Correction Procedure for Indirect Trajectory Optimization," Journal of Spacecraft and Rockets, Vol. 6, No. 9, September 1969, pages 1084-1086 by AIAA.